LC 2017: Paper 2

Question 4 (25 marks)

Question 4 (a)

Equation of a circle:
$x^2 + y^2 + 2gx + 2fy + c = 0$
Centre $= (-g, -f)$
$r = \sqrt{g^2 + f^2 - c}$

$A(0, 0)$, $B(6{\cdot}5, 0)$, $C(10, 7)$

$s : x^2 + y^2 + 2gx + 2fy + c = 0$

$A(0, 0) \in s : (0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0 \Rightarrow c = 0$

$B(6{\cdot}5, 0) \in s : (6{\cdot}5)^2 + (0)^2 + 2g(6{\cdot}5) + 2f(0) + (0) = 0$

$(6{\cdot}5)^2 + 2g(6{\cdot}5) = 0$

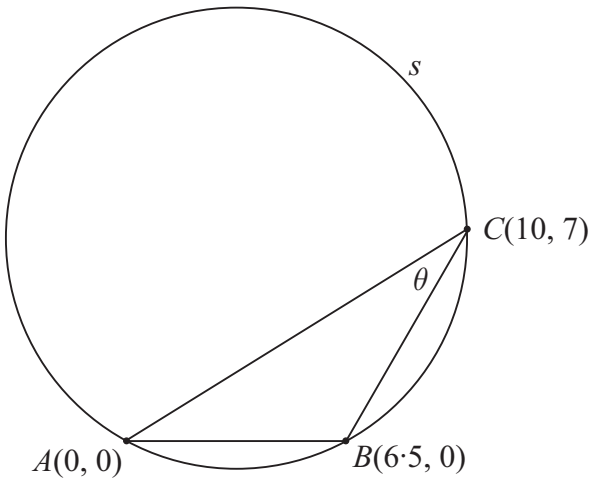
$(6{\cdot}5) + 2g = 0$

$g = -\frac{13}{4}$

$C(10, 7) \in s : (10)^2 + (7)^2 + 2(-\frac{13}{4})(10) + 2f(7) + (0) = 0$

$100 + 49 - 65 + 14f = 0$

$14f = -84 \Rightarrow f = -6$



$s : x^2 + y^2 + 2(-\frac{13}{4})x + 2(-6)y + (0) = 0$

$x^2 + y^2 - \frac{13}{2}x - 12y = 0$

$2x^2 + 2y^2 - 13x - 24y = 0$

Question 4 (b)

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$A(0, 0)$, $B(6{\cdot}5, 0)$, $C(10, 7)$

Slope of BC: $m_1 = \dfrac{7-0}{10-6{\cdot}5} = \dfrac{7}{3{\cdot}5} = 2$

Slope of AC: $m_2 = \dfrac{7-0}{10-0} = \dfrac{7}{10}$

$\tan\theta = \left|\dfrac{2 - \frac{7}{10}}{1 + 2(\frac{7}{10})}\right| \Rightarrow \theta = 28{\cdot}44^\circ$