

LC 2017 (SET A): PAPER 1**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$f(x) = \frac{1}{3}x^2 - x + 3$$

$$f(x+h) = \frac{1}{3}(x+h)^2 - (x+h) + 3 = \frac{1}{3}x^2 + \frac{2}{3}hx + \frac{1}{3}h^2 - x - h + 3$$

$$f(x+h) - f(x) = \frac{1}{3}x^2 + \frac{2}{3}hx + \frac{1}{3}h^2 - x - h + 3 - \frac{1}{3}x^2 + x - 3 = \frac{2}{3}hx + \frac{1}{3}h^2 - h$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) = \lim_{h \rightarrow 0} \left(\frac{\frac{2}{3}hx + \frac{1}{3}h^2 - h}{h} \right) = \lim_{h \rightarrow 0} \left(\frac{2}{3}x + \frac{1}{3}h - 1 \right) = \frac{2}{3}x - 1$$

Question 3 (b)

$$y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = \ln f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{f(x)} \times f'(x)$$

THE RULE IN WORDS: $y = \ln f(x) \Rightarrow \frac{dy}{dx} =$ one over the function inside the ln multiplied by the derivative of the function inside the ln.

$$f(x) = \ln(3x^2 + 2)$$

$$g(x) = x + 5$$

$$\begin{aligned} f(g(x)) &= \ln(3(x+5)^2 + 2) = \ln(3(x^2 + 10x + 25) + 2) \\ &= \ln(3x^2 + 10x + 25) + 2) \\ &= \ln(3x^2 + 30x + 75 + 2) \\ &= \ln(3x^2 + 30x + 77) \end{aligned}$$

$$[f(g(x))]' = \frac{1}{3x^2 + 30x + 77} \times 6x + 30 = \frac{6x + 30}{3x^2 + 30x + 77}$$

$$[f(g(\frac{1}{4}))]' = \frac{6(\frac{1}{4}) + 30}{3(\frac{1}{4})^2 + 30(\frac{1}{4}) + 77} = \frac{504}{1355} = 0.372$$
