

LC 2017: PAPER 1

QUESTION 2 (25 MARKS)

Question 2 (a)

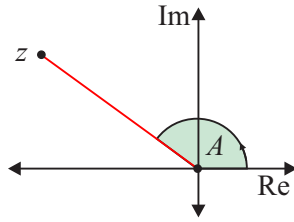
$$z = -\sqrt{3} + i$$

$$|z| = \sqrt{(-\sqrt{3})^2 + 1^2}$$

$$= \sqrt{3+1} = \sqrt{4} = 2$$

$$\theta = \tan^{-1} \left| \frac{1}{\sqrt{3}} \right| = \frac{\pi}{6}$$

$$\arg z = A = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ (Second quadrant)}$$



$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

$$z = 2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$z^4 = \left[2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \right]^4 = 2^4 \left(\cos \frac{20\pi}{6} + i \sin \frac{20\pi}{6} \right)$$

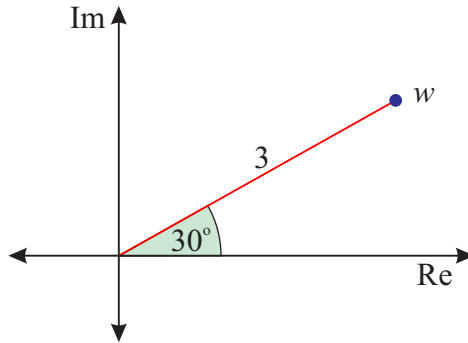
$$= 16 \left(\cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3} \right) = 16 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$

$$= -8 - 8\sqrt{3}i$$

Question 2 (b)

$$|w| = 3, \arg w = 30^\circ = \frac{\pi}{6}$$

$$w = 3 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$



$$t = zw = 2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \times 3 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$= 6 \left(\cos \left(\frac{5\pi}{6} + \frac{\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} + \frac{\pi}{6} \right) \right)$$

$$= 6(\cos \pi + i \sin \pi) = 6(-1) = -6$$
