LC 2017: Paper 2

Question 1 (25 marks)

Question 1 (a)

Success S: $p = \frac{1}{5} = 0 \cdot 2$

Failure F: $q = \frac{4}{5} = 0 \cdot 8$

$$\boxed{P(r) = {}^{n}C_{r}(p)^{r}(q)^{n-r}}$$

$$P(F, S, F, S, F, S, F) = 0 \cdot 8 \times 0 \cdot 2 \times 0 \cdot 8 \times 0 \cdot 2 \times 0 \cdot 8 \times 0 \cdot 2 \times 0 \cdot 8 = \frac{256}{78\,125}$$

Question 1 (b)

$$P(3S \text{ in first 6 trials and then } S \text{ in last trial}) = {}^{6}C_{3} \times (0 \cdot 2)^{3} \times (0 \cdot 8)^{3} \times (0 \cdot 2) = \frac{256}{15\,635}$$

Question 1 (c)

$$P(\text{At least one } S) = 1 - P(\text{No successes})$$

$$= 1 - {}^{n}C_{0} \times (0 \cdot 2)^{0} \times (0 \cdot 8)^{n} = 1 - 0 \cdot 8^{n}$$

Question 1 (d)

$$P(\text{At least one } S) = 1 - 0 \cdot 8^{n} = 0 \cdot 99$$

$$1 - 0 \cdot 99 = 0 \cdot 8^{n}$$

$$0 \cdot 01 = 0 \cdot 8^{n}$$

$$\log_{10} 0 \cdot 01 = \log_{10} 0 \cdot 8^{n}$$

$$\log_{10} 0 \cdot 01 = n \log_{10} 0 \cdot 8$$

$$n = \frac{\log_{10} 0 \cdot 01}{\log_{10} 0 \cdot 8} = 20 \cdot 63$$

Minimum value of $n = 21$