LC 2017 (Set A): Paper 1

Question 1 (25 marks)

Question 1 (a)

$f(x) = 2x^2 - 7x - 10 = 2(x^2 - \frac{7}{2}x - 5)$ $[k = -\frac{7}{2}]$

$= 2((x - \frac{7}{4})^2 - (-\frac{7}{4})^2 - 5)$

$= 2((x - \frac{7}{4})^2 - \frac{49}{16} - 5)$

$= 2((x - \frac{7}{4})^2 - \frac{129}{16})$

$= 2(x - \frac{7}{4})^2 - \frac{129}{8}$

$$\left(x + \frac{k}{2}\right)^2 = x^2 + kx + \left(\frac{k}{2}\right)^2$$

$$\therefore x^2 + kx = \left(x + \frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2$$

Question 1 (b)

Minimum value of $f(x)$ occurs when $(x - \frac{7}{4})^2 = 0 \Rightarrow x = \frac{7}{4}$

Minimum value occurs at $f(x) = -\frac{129}{8}$

Minimum point $= (\frac{7}{4}, -\frac{129}{8})$

Question 1 (c) (i)

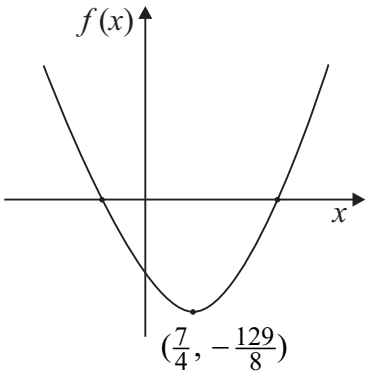
Two real roots: $b^2 - 4ac > 0$

$a = 2,\ b = -7,\ c = -10$

$(-7)^2 - 4(2)(-10) = 49 + 80 = 129 > 0 \Rightarrow f$ has two real roots

Or

As the coefficient of x^2 is positive the graph is shaped as shown to the right. The minimum point is below the x-axis giving two real roots.



Question 1 (c) (ii)

$f(x) = 2(x - \frac{7}{4})^2 - \frac{129}{8} = 0$

$2(x - \frac{7}{4})^2 = \frac{129}{8}$

$(x - \frac{7}{4})^2 = \frac{129}{16}$

$(x - \frac{7}{4}) = \pm\sqrt{\frac{129}{16}}$

$x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$